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Deriving Unbounded Reachability Proof of Linear Hybrid Automata during Bounded Checking Procedure

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Abstract—Reachability analysis of linear hybrid automata (LHA) is an important problem. Classical model checking (CMC) technique is not scalable and not guaranteed to terminate. On the other hand, bounded model checking (BMC) is more cost-effective to conduct but can not guarantee the safety beyond the bound. In this paper, we seek to bridge the gap between BMC and CMC for reachability analysis of LHA. During BMC of LHA, typical procedures can discover sets of unsatisfiable constraint cores, which can be mapped back to path segments in the graph structure of LHA. If every path connecting the initial and target location has to go through such infeasible path segment, the target location is entirely not reachable. Based on this characteristic, we propose a LTL model checking based approach to check whether the target location is blocked. To further optimize the performance, we propose an automata based solution to check the LTL specification incrementally and adopt an on-the-fly algorithm to check the accepting condition to avoid an explicit construction of product automata.

Index Terms—Reachability checking, linear hybrid automata, irreducible infeasible set, linear temporal logic, Büchi automata

1 INTRODUCTION

Hybrid Automata [1] are popular frameworks used to model hybrid systems that exhibit both continuous and discrete dynamic behavior. Reachability analysis of hybrid automata is considerably difficult. Even for a relatively simple class such as linear hybrid automata (LHA), the reachability problem is undecidable [2]. Classical model checking (CMC) techniques attempt to compute exact reachable state space of LHA by the expensive polyhedral based computation which is sensitive to the number of continuous variables and not guaranteed to terminate. Despite decades of research on it, state-of-the-art tools based on CMC technique such as HyTech [3], PHAVer [4] and PHAVer’s new implementation SpaceEx [5] do not scale well to problems of practical interest.

As an alternative technique to the classical symbolic model checking, bounded model checking (BMC) [6] was proposed to find bugs within a given threshold. The basic idea of BMC is to search for a counterexample with length no longer than an integer $k$ in model executions, when the complete behavior state space is too complex to check by classical model checking. The typical approach of BMC reachability analysis of LHA is to encode the state space of LHA within the bound into a constraint set which can then be solved with Satisfiability Modulo Theories (SMT) solvers [7], [8]. However, as the technique needs to encode the whole state space within the bound first, the high complexity restricts the size of the system that can be handled.

The past decade has witnessed a significant amount of progress in constraint solving technologies, thanks to the emergence of highly efficient SMT solvers [7], [8], the size of the BMC problem of LHA that can be verified has increased significantly. However, the result of BMC verification only covers bounded behavior of the model. It remains a very interesting problem that whether we can get an unbounded proof of the system from a BMC procedure. In finite-state BMC, techniques such as $k$-induction [21], interpolation [28], IC3 [23] are widely used to obtain an unbounded model checking result. Nevertheless, they are difficult to conduct on infinite state systems like hybrid system.

Another way to solve the BMC problem of LHA is a path-oriented style approach [9]. The basic idea is to check one abstract path in the graph structure of an LHA at one time using Linear Programming (LP) to find whether there exists a feasible continuous behavior of the LHA along with this discrete path. As the number of paths within a given threshold is finite, we can answer the BMC problem of LHA by enumerating and checking all the candidate paths one by one [10]. Furthermore, when a path is proved to be infeasible, irreducible infeasible set (IIS) technique [12] can be deployed on the constraint set generated according to the path under checking to retrieve a minimal inconsistent constraint set. Such inconsistent constraint set can be mapped back to a path segment in the graph structure of LHA, which is infeasible for sure. As any path containing an infeasible path segment is definitely infeasible, the infeasible path segments, IIS, located by the underlying LP solver can be utilized to accelerate the BMC solving process [11], [16].
In the above path-oriented BMC approach, as any candidate path has to avoid known infeasible path segments, such path segments can also be used to block the graph structure of LHA. During the experiment, it is not rare to see that after several path segments are blocked, the path-enumerating procedure cannot find any discrete path to reach the target location without touching any of the previously detected IIS path segments in the LHA model. In this situation, there does not exist a continuous behavior to reach the target location entirely, not only within the bound.

Based on this observation, we propose to exploit IIS path segments to prove whether a bounded unreachable argument can be extended to the unbounded state space. We propose a linear temporal logic (LTL) [18], [19] based sound but not complete approach to tackle such problem. In our approach, the discrete part of the LHA model, the graph structure is presented as a finite state transition system [10]. Then, we encode all the IIS path segments detected during BMC solving into an LTL formula. By checking whether the transition system satisfies the LTL formula, we can prove whether there exists a discrete path to reach the target location without containing any known IIS path segment in the graph structure of the LHA. If the LTL specification is satisfied, it means there will not exist such a path no matter how large the bound is given, which indicates that the bounded unreachable statement also stands in the complete unbounded state space.

A basic solution of integrating the BMC and proof procedure is to conduct the LTL based proof after the completion of the BMC analysis. It works as follows: Once an LHA model and a reachability specification are given, we start to perform the path-oriented bounded reachability analysis. When the BMC procedure finishes and reports the target is not reachable in the given bound, all the IIS path segments detected during the BMC solving process are encoded into an LTL formula. Then, a state-of-the-art LTL model checker is deployed to check whether the LTL specification is satisfied by the transition system constructed from the graph structure of LHA model. If yes, an unbounded result is proved, otherwise a bounded result is given.

In the above approach, as the LTL checking is conducted after the BMC procedure finishes, they do not have any interaction. When the LTL specification can be proved at an earlier time, it is a waste of time to continue the BMC analysis. In order to integrate the BMC and the proof procedure tightly to achieve better performance, we propose to perform the LTL checking whenever a new IIS path segment is located during the BMC procedure. In this way, the proof procedure will stop the BMC procedure as soon as the LTL specification becomes satisfied.

Although the tight integration can avoid meaningless BMC checking in some cases, it has to conduct repeated LTL model checking, in other words, call a third-party checker frequently, which incurs a high overhead. In order to solve this issue, we propose to check the LTL specification incrementally by constructing an equivalent Büchi automata. With the help of a dedicated LTL-to-Büchi translation algorithm introduced in [24], we can generate a small Büchi automata efficiently. Furthermore, we propose an on-the-fly algorithm to check the emptiness of the product transition system in the final step of LTL model checking. In this way, we can frequently construct only a small portion of the state space before a counterexample of the property being checked is found.

The above mentioned LTL and automata based approaches have been implemented into a bounded LHA checker BACH [10]. To evaluate the performance of our approach, we conduct a series of case studies on the enhanced BACH, and compare it with the state-of-the-art classical model checker, SpaceEx [5]. The set of experiment results shows that most of the benchmarks can be proved to be unreachable in the unbounded state space by analyzing the intermediate results of the BMC procedure without performing the expensive classical model checking. Furthermore, the experiments also show the performance of the approach presented in this paper outperforms the state-of-the-art CMC checker, SpaceEx, significantly.

2 Path-Oriented Reachability Analysis

In this section, we give the formal definition of linear hybrid automata and recap the underlying technique of path-oriented bounded reachability analysis that was proposed in [9], [10]. Furthermore, we present our method of using irreducible infeasible set technique to locate infeasible path segment from a path which is proved to be infeasible by the path-oriented checking to accelerate the BMC procedure [16].

2.1 Basic Path-Oriented Reachability Analysis

Definition 1. The LHA considered in this paper is defined as a tuple \( H = (G, X, \alpha, \beta, \phi, \psi) \), where
\[
G = (Q, q_0, \text{bad}, \Sigma, E) \quad \text{is the (labeled) location graph of } H, \text{where}
\]
\[Q \text{ is a finite set of locations;}
\]
\[\alpha \in Q \text{ is initial location;}
\]
\[\text{bad} \subset Q \text{ is bad location (the location that should not be reachable);}
\]
\[\Sigma \text{ is a finite set of labels;}
\]
\[E \subset (Q \setminus \text{bad}) \times \Sigma \times Q \text{ is a finite set of (labeled) transitions, where no two outgoing transitions from a given location have the same label;}
\]
\[X \text{ is a finite set of state continuous variables.}
\]
\[\alpha \text{ is a labeling function which maps each location in } Q \setminus \{q_0\} \text{ to a set of location invariants which are of the form } a \leq \sum_{i=0}^l c_i x_i, \text{ where } c_i \in \{a, b, c\} \text{ are real numbers (} a, b \text{ may be } -\infty).\n\]
\[\beta \text{ is a labeling function which maps each location in } Q \setminus \{q_0\} \text{ to a set of flow conditions which are of the form } \bar{x} \in [a, b], \text{ where } x \in X, \text{ and } a, b \text{ are real numbers (} a \leq b).\n\]
\[\phi \text{ is a set of labeling functions which map each transition in } E \text{ to a set of transition guards which are of the form } a \leq \sum_{i=0}^l c_i x_i, \text{ where } x_i \in X, \text{ and } a, b \text{ are real numbers (} a, b \text{ may be } -\infty).\n\]
\[\psi \text{ is a set of labeling functions which map each transition in } E \text{ to a set of reset actions which are of the form } x := c, \text{ where } x \in X, c \text{ is a real number.}\n\]

We use sequences of locations to represent the evolution of an LHA from location to location. For an LHA \( H = (G, X, \alpha, \beta, \phi, \psi) \), a path segment is a sequence of locations of the form \( (v_0)_{\sigma_0} (v_1)_{\sigma_1} \cdots (v_n)_{\sigma_n} \), which satisfies \( (v_i, \sigma_i, v_{i+1}) \in E \) for each \( i (0 \leq i < n) \),
where $\sigma_t \in \Sigma, \phi_i \in \phi, \psi_i \in \psi$. A path in $H$ is a path segment starting from the initial location $q_0$.

For a path in $H$ of the form $\langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$, by assigning each location $v_i$ with a time delay stamp $\delta_i$, we get a timed sequence of the form $\langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$ where $\delta_i (0 < i \leq n)$ is a nonnegative real number and $\delta_0 = 0$ as $v_0 = q_0$ is initial location. This time sequence represents a behavior of $H$ such that the system starts from the initial location $v_0$, stays there for $\delta_0$ time units, which is 0, then jumps to $v_1$ and stays for $\delta_1$ time units, and so on.

The behavior of an LHA can be described informally as follows. The automaton jumps from initial location $v_0$ to $v_1$ to initialize all the variables. Then, as time progresses, the values of all variables change continuously according to the flow conditions associated with the current location. At any time, the system can change its current location from $v_i$ to $v_{i+1}$ provided that there is a transition $(v_i, \sigma_i, v')$ from $v_i$ to $v'$ whose all transition guards are satisfied by the current values of the variables. With a location changed by a transition $(v_i, \sigma_i, v')$, some variables are reset to the new value accordingly to the resets in $\psi$. Transitions are assumed to be instantaneous.

Let $H = (G, X, \alpha, \beta, \phi, \psi)$ be an LHA. Given a timed sequence $\omega$ of the form $\langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$, let $\xi_i(x)$ represents the value of $x (x \in X)$ when the automaton has stayed at $v_i$ for delay $\delta_i$ and $\lambda_i(x)$ represents the value of $x$ at the time the automaton reaches $v_i$ along with $\omega (0 \leq i \leq n)$. It follows that $\lambda_{i+1}(x) =$

$$
\begin{cases} 
\delta_i & \text{if } x := d \in \psi_i (0 \leq i < n), \\
\xi_i(x) & \text{otherwise}
\end{cases}
$$

**Definition 2.** For an LHA $H = (G, X, \alpha, \beta, \phi, \psi)$, a timed sequence of the form $\langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$ represents a behavior of $H$ if and only if the following condition is satisfied:

- Each variable $x \in X$ evolves according to its flow condition in each location $v_i (0 < i \leq n)$, i.e., $u_i \leq \xi_i(x) - \lambda_i(x) \leq u'_i$, where $x \in \{u_i, u'_i\} \in \beta(v_i)$;
- All the transition guards in $\phi_i (1 \leq i \leq n-1)$ are satisfied, i.e., for each transition guard $a \leq \sum_{k=0}^{l} c_k \xi_k(x_k) \leq b$ in $\phi_i, a \leq \sum_{k=0}^{l} c_k \xi_k(x_k) \leq b$;
- The location invariant of each location $v_i (1 \leq i \leq n)$ is satisfied, i.e., at the time the automaton reaches and leaves $v_i$, each constraint $a \leq \sum_{k=0}^{l} c_k \lambda_k(x_k) \leq b$ in $\alpha(v_i)$ $(1 \leq i \leq n)$ is satisfied, i.e., $a \leq \sum_{k=0}^{l} c_k \lambda_k(x_k) \leq b$ and $a \leq \sum_{k=0}^{l} c_k \lambda_k(x_k) \leq b$.

**Definition 3.** For an LHA $H = (G, X, \alpha, \beta, \phi, \psi)$, if a timed sequence of the form $\langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$ is a behavior of $H$, we say path $\rho = \langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$ is feasible, and location $v_n$ is reachable along $\rho$.

For an LHA $H = (G, X, \alpha, \beta, \phi, \psi)$, a reachability specification, denoted as $R(v, \varphi)$, is a location $v$ in $H$ and a set $\varphi$ of variable constraints of the form $a \leq \sum_{i=0}^{m} c_i x_i \leq b$, where $x_i \in X$ for any $i (0 \leq i \leq l)$, $\alpha, \beta$ and $c_i (0 \leq i \leq l)$ are real numbers, $(\alpha, \beta) = (\text{minus}) \in$. 

**Definition 4.** Let $H = (G, X, \alpha, \beta, \phi, \psi)$ be an LHA, and $R(v, \varphi)$ be a reachability specification. A behavior of $H$ of the form $\langle v_0 \rangle (\phi_0, \psi_0)_{\sigma_0} (v_1) (\phi_1, \psi_1)_{\sigma_1} \cdots (\phi_{n-1}, \psi_{n-1})_{\sigma_{n-1}} (v_n)$ satisfies $R(v, \varphi)$ if and only if for each constraint in $\varphi$ is satisfied when the automaton has stayed at $v_n$ for delay $\delta_n$, i.e., for each variable constraint $a \leq \sum_{i=0}^{m} c_i x_i \leq b$ in $\varphi$, $a \leq \sum_{k=0}^{l} c_k \lambda_k(x_k) \leq b$ where $\lambda_k(x_k) (0 \leq k \leq l)$ represents the value of $x_k$ when the automaton stays at $v_i$. For the delay $\delta_n$, $H$ satisfies $R(v, \varphi)$ if and only if there is a behavior of $H$ which satisfies $R(v, \varphi)$.

According to Definitions 2 and 4, the reachability verification problem can be translated into the feasibility problem of a set of constraints on variables $\lambda_i(x)$, and $\xi_i(x)$ where $x \in X$ could consume quite a lot of time. Fortunately, as we use LP to judge the feasibility of a timed sequence, the number of candidate paths with length no longer than $k$ is finite. Therefore, if we enumerate and check all the paths in the bound one by one, the bounded reachability problem can be tackled in the path-oriented BMC way.

### 2.2 IIS Based BMC Acceleration

Last paragraph gives a simple solution of the path-oriented bounded reachability analysis of LHA, which requires to enumerate and check all the candidate paths one by one in the graph structure of LHA. Nevertheless, when the given threshold is large and the graph structure of LHA is complex, there would be numerous paths to traverse, which could consume quite a lot of time.

 Fortunately, as we use LP to judge the feasibility of a path $\rho$, irreducible infeasible set [12] technique can be deployed to locate a minimal infeasible set of the linear constraint set w.r.t. $\rho$.

Generally speaking, a set of linear constraints $C$ is said to be satisfiable, if there exists a valuation of all the variables, which makes all the constraints in $C$ to be true. Otherwise, $C$ is unsatisfiable. If $C$ is unsatisfiable, then IIS of $C$ is a subset $C' \subseteq C$ that $C'$ is unsatisfiable and for any $C'' \subseteq C'$, $C''$ is satisfiable.
Intuitively speaking, an IIS of a linear constraint set is an unsatisfiable set of constraints that becomes satisfiable if any constraint is removed. Therefore, given an infeasible path \( \rho \), we can analyze the constraint set \( C \) generated according to this path to locate an IIS \( C' \). As each constraint in \( C \) is generated from a semantical element, e.g., location invariants or transition guards, in the locations and transitions from the model. Therefore, for each constraint in \( C \), we can locate the source location or transition in a straightforward way. As a result, the constraint set located by the IIS technique can be mapped back to a path segment \( \gamma \) in \( \rho \). In other words, once a path is proved to be infeasible, an infeasible path segment can be located from it by IIS analysis.

Fortunately, quoted from [12], the algorithm to locate the IIS from a unsatisfiable set is “simple, relatively efficient and easily incorporation into standard LP solvers”. Many software packages are available, which supports the efficient analysis of a linear constraint set and locating of the minimal IIS, such as IBM CPLEX [30] and LINDO [31].

Now, let’s see a path \( \rho = \langle v_0 \rangle \xrightarrow{e_0} \langle v_1 \rangle \xrightarrow{e_1} \langle v_2 \rangle \xrightarrow{e_2} \langle v_3 \rangle \xrightarrow{e_3} \langle v_4 \rangle \xrightarrow{e_4} \langle v_5 \rangle \) in the Water-Level Monitor System (Fig. 1), which is proven to be infeasible. The IIS of the constraint set according to \( \rho \) given by the underlying LP solver is \( C'_\rho = \{ \delta_{e_1} \geq 0, \xi_{v_1}(x) - \lambda_{v_1}(x) = \delta_{e_1}, \xi_{v_4}(y) - \lambda_{v_4}(y) = -2 \delta_{e_4}, \lambda_{v_2}(y) = \xi_{v_2}(y), \xi_{v_3}(y) - \lambda_{v_3}(y) = \delta_{e_3}, \lambda_{v_1}(x) = 0, \lambda_{v_4}(y) = 5, \xi_{v_1}(x) = 2, \xi_{v_2}(y) = 0 \} \), where \( e_1 \) represents the second occurrence of the location \( v_1 \) in \( \rho \) (the 6th location).

Now we show how can this IIS be mapped back to a path segment in \( \rho \).

- \( \delta_{e_1} \geq 0 \) stands for the time elapsed in location \( v_1 \) is nonnegative.
- \( \xi_{v_1}(x) - \lambda_{v_1}(x) = \delta_{e_1}, \xi_{v_4}(y) - \lambda_{v_4}(y) = -2 \delta_{e_4}, \lambda_{v_2}(y) = \xi_{v_2}(y), \xi_{v_3}(y) - \lambda_{v_3}(y) = \delta_{e_3}, \lambda_{v_1}(x) = 0, \lambda_{v_4}(y) = 5, \xi_{v_1}(x) = 2, \xi_{v_2}(y) = 0 \), come from the flow conditions of \( \dot{x} \) and \( y \) in location \( v_1: \dot{x} = 1, \dot{y} = -2 \), and location \( v_2: \dot{y} = 1 \).
- \( \lambda_{v_1}(x) = 0 \) and \( \lambda_{v_4}(y) = 5 \) come from the transition guard and reset action on transition \( e_1: y = 5, x = 0 \).
- \( \xi_{v_3}(x) = 2 \) and \( \xi_{v_2}(y) = 0 \) come from the transition guards on transition \( e_3: x = 2 \) and \( e_4: y = 0 \).
- \( \lambda_{v_2}(y) = \xi_{v_2}(y) \) comes from the transition guard on transition \( e_1 \) as \( y \) is not reset on \( e_1 \).

Therefore, the related locations and transitions of \( C'_\rho \) include \( v_1, v_2, e_1, e_3, e_4, e_5 \). As the corresponding infeasible path segment \( \rho' \) of \( C'_\rho \) should be the shortest path segment which contains all these elements in \( \rho \), it is \( \langle v_3 \rangle \xrightarrow{e_3} \langle v_4 \rangle \xrightarrow{e_4} \langle v_5 \rangle \).

Clearly, a path \( \rho \) can be falsified for verification without calling the underlying decision procedure if it contains an infeasible path segment \( \rho' \), since the occurrence of \( \delta \) in \( \rho \) will just be translated into the same set of unsatisfiable constraints with only name changed. Therefore, the path-encoding procedure should rule out paths containing any known infeasible path segment.

For each bound \( k \), the workflow of the IIS based path-oriented BMC solution for LHA is shown in Fig. 2 [16]. First, a potential path \( \rho \) with length no longer than \( k \) is enumerated and then analyzed by an LP solver. If \( \rho \) is feasible, the algorithm terminates and reports \( \rho \) as a witness, otherwise IIS technique is deployed to locate an infeasible path segment from \( \rho \), which will be fed back to the path-traversing procedure to accelerate the BMC process. The algorithm terminates when no more candidate path can be found or a counterexample is confirmed.

Consider the automaton in Fig. 1, suppose we want to check whether location \( v_5 \) is reachable along a path with...
bound 20 and the first enumerated path is $\rho_1 = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_4 \rangle \rightarrow \langle v_5 \rangle$. The underlying LP solver proves $\rho_1$ is infeasible and locates an IIS path segment $\rho'_1 = \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_4 \rangle \rightarrow \langle v_5 \rangle$, which will be fed back to the path-enumerating procedure to block such a path segment. Then the next found potential path to reach $v_5$ without containing $\rho'_1$ in the graph is $\rho_2 = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_4 \rangle \rightarrow \langle v_5 \rangle$. Also, it is infeasible and the IIS path segment $\rho'_2 = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_5 \rangle$ which is the path itself. The third candidate path to reach $v_5$ is $\rho_3 = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_4 \rangle \rightarrow \langle v_5 \rangle \rightarrow \langle v_6 \rangle$. Nevertheless, we do not need to check this path by LP since it contains an exact IIS path segment $\rho'_3$. After the two IIS path segments $\rho'_1$ and $\rho'_2$ are blocked, no candidate path within the given bound can be found, which implies the target location is not reachable within the given step threshold.

3 DERIVING UNBOUNDED RESULT DURING BMC SOLVING PROCESS

3.1 Motivation Overview

The last section gives a quick review of the path-oriented bounded reachability analysis of LHA [16]. The basic idea is to find an abstract path, sequence of locations, in the discrete graph structure of an LHA under verification first. Then, we can check whether there exists a feasible continuous behavior corresponding to certain discrete path. During the procedure, IIS technique is deployed to locate infeasible path segments in the graph model. Such path segments can be utilized to tailor the bounded graph structure of the LHA model under verification to accelerate the BMC solving process.

Clearly, the path segment w.r.t. each IIS is infeasible for sure. In other words, once a new IIS is found, a path segment in the graph structure of the LHA model under verification can be blocked. During experiments, we can see this scenario very often: an LHA model under verification does not have any path left after several IIS path segments are blocked. In this situation, the reachability specification can not be satisfied at all, not only in the bound. Consider the previous example in Fig. 1, the detected IIS path segments are $\rho'_1 = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_5 \rangle$ and $\rho'_2 = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \langle v_2 \rangle \rightarrow \langle v_3 \rangle \rightarrow \langle v_5 \rangle$, and the target location is $v_5$. As we can see from the graph, if we block these two path segments in the graph structure of the model, there will not exist any path to reach $v_5$ anymore.

Based on the observation, if we can prove that there does not exist any path to reach the target location without going through certain infeasible path segments in the graph structure of an LHA model, we can tell that the reachability specification is not satisfied in the complete unbounded state space. In other words, the problem becomes as follows: We have a directed graph with an initial location and a target location, A set of path segments in the graph are blocked. Then, whether any path connecting the initial and target location exists in the directed graph?

3.2 LTL Based IIS Representation and Verification

The question raised in the end of the last paragraph concerns the reachability problem only on the discrete level, the graph structure $G$, of the LHA model $H$. In order to conduct reachability verification on $G$, we propose to extend $G$ into a typical transition system (TS) [19] $T$ first.

Definition 5. Given an LHA $H = (G, X, \alpha, \beta, \phi, \psi)$, the related discrete transition system (DTS) of its graph structure $T = \{G', AP, L\}$:

- $G' = (Q, q_0, q_{had}; \Sigma, E')$ is an extension to the (labeled) location graph $G$, where
  - $Q, q_0, q_{had}; \Sigma \subseteq G$;
  - $E' \subseteq Q \times \Sigma \times Q$ is a finite set of (labeled) transitions. $E' = E \cup \epsilon$, where $E \subseteq G, \epsilon$ is a self-loop starting and ending in location $q_{had}$.
- $AP$ is the atomic proposition set in $T$. For each $q_i \in Q$, there exists an atomic proposition $p_{q_i} \in AP$.
- $L : Q \rightarrow 2^\mathbb{P}$ is a labeling function. For each $q_i \in Q$, $L(q_i) = \{p_{q_i}\}$.

Let us review the LHA presented in Fig. 1, the DTS $T$ modeling the graph structure of this LHA model is shown in Fig. 3.

It is well known that linear temporal logic [18] is a powerful temporal logic for describing system behaviors. Hence, we propose to use LTL to describe the property that there exists a path in the TS to reach the target location $q_{had}$ without containing any previously detected IIS path segment.

Avoiding IIS Path Segment. The first thing we need to do is to represent the IIS path segment in LTL. Suppose there is a path $\rho = \langle v_0 \rangle \rightarrow \langle v_1 \rangle \rightarrow \cdots \rightarrow \langle v_n \rangle$ in an LHA $H$, and $\rho$ contains a path segment $\rho' = \langle v_i \rangle \rightarrow \langle v_{i+1} \rangle \rightarrow \cdots \rightarrow \langle v_k \rangle (i \geq 0$ and $j \leq n)$. As $T$ shares the same graph structure $G$ with $H$, $\rho$ and $\rho'$ are also paths in the DTS $T$ w.r.t. $H$. According to Definition 5, we have $L(v_i) = p_{v_i}, (0 \leq i \leq k)$. Therefore, $L(v_i) = p_{v_0}, \ldots, L(v_i) = p_{v_i}, \ldots, L(v_k) = p_{v_{k}}$, accordingly.

As $\rho'$ starts from location $v_i$, $v_i$ satisfies LTL formula $p_{v_i} \land X p_{v_{i+1}} \land \cdots \land X X \cdots X p_{v_k}$. Based on this, given an IIS path segment $\rho' = \langle v_i \rangle \rightarrow \langle v_{i+1} \rangle \rightarrow \cdots \rightarrow \langle v_k \rangle$, we give the LTL formula representing this path segment as

$$IIS_{\rho'} = p_{v_i} \land X p_{v_{i+1}} \land \cdots \land X X \cdots X p_{v_k}.$$ 

Furthermore, any path which does not contain path segment $\rho'$ satisfies LTL formula $G(-IIS_{\rho'})$.

1. Note that this discrete transition system only focuses on the discrete graph structure of LHA. It is different from the one used to specify the semantics of LHA.
Reaching Target Location without Containing IIS Path Segment. The property that target location \(q_{\text{bad}}\) can be reached by a path in the graph structure of an LHA can be simply represented by an LTL formula \(F p_{q_{\text{bad}}}\). Then, given a set of IIS path segments \(\{\rho_1, \rho_2, \ldots, \rho_n\}\), the LTL formula which is true for a path reaching the target location without containing any above IIS path segment is shown below

\[
(G(\bigwedge_{1\leq i\leq n} \neg\text{IIS}_{\rho_i})) \land F p_{q_{\text{bad}}},
\]

where \(\text{IIS}_{\rho_i}\) represents the \(i\)th IIS path segment. As our target is to prove the nonexistence of such a path, the final LTL specification is the negation of the above formula

\[
(\neg(G(\bigwedge_{1\leq i\leq n} \neg\text{IIS}_{\rho_i})) \land F p_{q_{\text{bad}}}).
\]

**Theorem 1.** Given an LHA \(H\), a target location \(q_{\text{bad}}\) and a set of IIS path segments \(\{\rho_1, \rho_2, \ldots, \rho_n\}\), if the DTS model \(T\) w.r.t. \(H\) satisfies the LTL specification \((G(\bigwedge_{1\leq i\leq n} \neg\text{IIS}_{\rho_i})) \land F p_{q_{\text{bad}}}\), then \(q_{\text{bad}}\) is not reachable in the complete state space of \(H\).

**Proof [Proof by Contradiction].** Assume \(T\) satisfies the LTL specification and \(q_{\text{bad}}\) is reachable in \(H\) along a path \(\rho = (v_0) \rightarrow (v_1) \rightarrow \cdots \rightarrow (v_n)(v_n = q_{\text{bad}})\). Clearly, as \(T\) and \(H\) shares the same graph structure \(G\), \(\rho\) is also a path in \(T\).

As \(q_{\text{bad}}\) is reachable along \(\rho\), this means there exists a feasible continuous behavior of \(H\) along \(\rho\), therefore \(\rho\) doesn’t contain any IIS path segments related with \(\{\rho_1, \rho_2, \ldots, \rho_n\}\) for sure. As a result, the LTL formula \((G(\bigwedge_{1\leq i\leq n} \neg\text{IIS}_{\rho_i})) \land F p_{q_{\text{bad}}}\) is true for \(\rho\). As we can see, \(\rho\) is the counterexample of the given LTL specification \((\neg(G(\bigwedge_{1\leq i\leq n} \neg\text{IIS}_{\rho_i})) \land F p_{q_{\text{bad}}})\). This contradicts the assumption that \(H\) satisfies the LTL specification. [Hence, the supposition is false and the proposition is true.]

Now, let us go back to the LHA in Fig. 1. As mentioned in Section 2, it has two IIS path segments: \((v_0) \rightarrow (v_1) \rightarrow (v_2)\), and \((v_4) \rightarrow (v_3) \rightarrow (v_1) \rightarrow (v_5)\) and the target location is \(v_5\). According to the above encoding method, the associated LTL specification is

\[
(\neg(G(\neg(p_0 \land X p_1 \land X X p_5)) \land \neg(p_4 \land X p_3 \land X X p_5))) \land F p_{v_5}.
\]

It is worth noting that in Definition 1, we only allow one transition between any two locations. In this case, there is no need to encode the edge in the path segment into our encoding, and thus we can give a simplified encoding solution. In the supplementary material, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TC.2016.2604308, we show that in the general case, when multiple transitions are allowed between two locations, our approach can also work by encoding edges into the corresponding LTL formula.

**3.3 Lazy Proof**

Model checking of LTL property on a transition system is a well-studied problem and there are various efficient tools such as Spin [20], NuSMV [17] to tackle the problem. Therefore, we can take advantage of these off-the-shelf LTL checkers to prove the reachability of certain target location in the discrete graph structure efficiently.

Recently, IC3 [23] algorithm was proposed to perform SAT-based model checking without unfolding the transition relation. It proved to outperform the existing verification techniques for finite-state systems in the last years. With the help of the converting tool smvtoaiger [27], we can formulate a LTL model checking problem with AIGER [27] standard which is widely used in hardware verification. Afterwards, we can also solve the problem efficiently with tools based on IC3 technique, such as timed [26].

Now, by using the mature checkers mentioned above to conduct LTL verification on the discrete transition graph of LHA model, we can prove that after certain path segments in the graph structure of the LHA model are blocked, whether there still has any potential path left to reach the target location. Clearly, if there does not exist a candidate path at all, then the target location is not reachable thoroughly. This provides a procedure that allows us to derive, in some cases a proof of unbounded result from a BMC solving process. As the LTL specification checking is conducted after the completion of BMC checking procedure, we call it Lazy Proof.

**Workflow of Lazy Proof.** Given an LHA model and a bound \(k\), the workflow of lazy proof is shown in Fig. 4. The path-oriented BMC is conducted first and all the IIS path segments detected during the procedure are saved for the follow-up proof procedure. The BMC solving process is marked as “BMC Procedure” by double square and its detailed flow is shown in Fig. 2 previously. After the BMC analysis completes, if a feasible path is found then it is reported as a witness which can confirm the target location is reachable. Otherwise, all the IIS path segments identified in the BMC procedure are encoded into an LTL specification. After that, an state-of-the-art LTL model checker is
called to perform the unbounded proof by checking whether the specification is true on the DTS of the LHA model. If yes, then the target location is not reachable completely, otherwise a $k$-bounded unreachable conclusion will be given. In another word, this approach is sound but not complete.

### 3.4 Eager Proof

In the above paragraph, we show a basic solution to integrate the path-oriented BMC and LTL-based proof procedure. It attempts to derive an unbounded reachability result by collecting and analyzing the intermediate results, IIS path segments, of the BMC solving process. As the proof procedure has to wait for the completion of the BMC procedure and it can not give feedback to the BMC procedure, the lazy proof based solution is not as efficient as it could be.

Let us consider the following scenario: ten IIS path segments were located after the BMC procedure finishes. However, the first two IIS path segments are able to block the target location already. In this case, it will be a waste of time to continue the BMC analysis after the first two IIS path segments are located. In fact, if we can prove the target location is not reachable already, we can stop the BMC procedure in advance.

In order to solve the problem, we attempt to combine the LTL checking and BMC procedure tightly, to check the LTL specification during the BMC solving process. Different from the lazy proof, which starts the LTL proof after the completion of BMC procedure, we call the new approach Eager Proof.

**Workflow of Eager Proof.** We can adapt the lazy proof to eager proof by modifying its workflow slightly. As we want to prove the LTL specification whenever a new IIS is detected, the BMC solving procedure is not treated as a black box again. The updated workflow is shown in Fig. 5. The main change is the time point of checking the satisfiability of the LTL specification. In the eager LTL-based approach, we call a mature LTL checker to check the LTL specification whenever a new IIS is detected during the BMC procedure. Besides, if the LTL checker finds the specification becomes satisfied, the BMC procedure will be stopped immediately and an unbounded result will be given.

### 3.5 Eager Automata-Based Proof

#### 3.5.1 Incremental Büchi Construction

In the above paragraphs, we introduce two LTL based proof strategies to derive an unbounded result from a BMC procedure. The lazy LTL-based solution may waste time to do meaningless checking when an unbounded proof can be derived in an earlier time. In order to solve this problem, an eager LTL-based solution is proposed to check the LTL specification once an IIS is detected. However, the eager solution needs to call the third party LTL checker to check the LTL specification repeatedly while the lazy solution only checks the specification once. When the LTL specification is hard to prove or can not be proved at all, the overhead caused by repeated LTL model checking would slow down the whole analysis greatly.

The basic idea of LTL model checking is to construct an equivalent büchi automata from the LTL formula first and then find a strong connected component (SCC) containing the accepting state in it. In our problem, we will detect more and more IIS path segments as BMC procedure continues, which means the LTL specification is refined gradually. Inspired by the two facts, we decide to check the LTL specification by constructing a büchi automata incrementally during the BMC solving process instead of calling a third party checker.

The LTL specification described in Section 3.2 has the following form: $\neg((G(\bigwedge_{1 \leq i \leq n} \neg IIS_{p_i})) \land F p_{\text{bad}})$. In the corresponding büchi, we need to search for a counterexample path that satisfies LTL formula $G(\bigwedge_{1 \leq i \leq n} \neg IIS_{p_i}) \land F p_{\text{bad}}$. We decide to divide it into two parts: $G(\bigwedge_{1 \leq i \leq n} \neg IIS_{p_i})$ and $F p_{\text{bad}}$ and search for the counterexample path in two stages. The reason is that the first part changes as more and more IIS path segments are detected while the second part remains the same all the time. It makes it easier to construct a büchi automata from LTL formula $G(\bigwedge_{1 \leq i \leq n} \neg IIS_{p_i})$ incrementally.

We could apply an existing LTL-to-büchi translation algorithm such as [25] to construct a büchi automata. However, since our LTL formula has a special structure, we can apply a dedicated incremental algorithm introduced in [24] to generate a very small büchi automata efficiently.

The workflow of automata based approach is shown in Fig. 6. Given an LHA and a bound $k$, we first construct a transition system $TS$ from the graph structure of LHA using commonly used method. Then we start the BMC procedure to enumerate candidate paths in the graph structure and check

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2. Due to the space limit, please refer to [24] for detail of the algorithm to construct a büchi automata.
whether it is feasible according to the semantic information along the path. If feasible, then we find a counterexample, report reachable; otherwise an IIS path segment is located and passed to the proof procedure to construct an IIS buchi automata. The IIS buchi automata is constructed incrementally according to the LTL formula $G(V_{1}/C_{2}0_{i}/C_{2}0_{n})$: IIS $r_i$ to rule out all the located IIS path segment.

After that we will intersect TS with the IIS buchi automata and then look for a path satisfying $F_{q_{bad}}$ in the product transition system. If such path does not exist, the original LTL specification is satisfied. Then the checking process will terminate immediately and give an unbounded result. Otherwise, the BMC procedure continues. New IIS path segment will be located and the IIS buchi automata will be adapted incrementally using the algorithm introduced in study [24] to rule out all the located IIS path segment.

After that we will intersect TS with the IIS buchi automata and then look for a path satisfying $F_{q_{bad}}$ in the product transition system. If such path does not exist, the original LTL specification is satisfied. Then the checking process will terminate immediately and give an unbounded result. Otherwise, the BMC procedure continues. New IIS path segment will be located and the IIS buchi automata will be adapted incrementally using the algorithm introduced in study [24] to rule out all the located IIS path segments. The whole analysis will terminate when an unbounded result is proved or a given bound is reached.

### 3.5.2 On-the-Fly Emptiness Checking

As the LTL property we want to verify in the second stage is quite simple, we propose to check it on the fly, which can guide the construction of the product transition system while computing the intersection of the graph TS and IIS buchi. In this way, we may frequently construct only a small portion of the state space before we find a counterexample to the property being checked. The algorithm is shown in Algorithm 1: we start from the initial location of the product transition system and search for a cycle containing $q_{bad}$ using the double depth first search (DFS) algorithm introduced in [35].

**Algorithm 1. On-the-Fly Emptiness Checking**

**Input:** graph transition system $TS$, IIS buchi automata $B$

**Output:** False if $TS \otimes B \models F_{q_{bad}}$, True otherwise.

**procedure** check($TS, B$)

$p = TS$.init
$q = B$.init

return dfs1($p, q$)

**end procedure**

**procedure** dfs1($p, q$)

hash($p, q$)

for all successors $p_1$ of $p$ do
  for all successors $q_1$ of $q$ with label $L(p_1)$ do
    if not accept($q_1$) then continue
    end if
    if $p_1 == q_{bad}$ then
      return not dfs2($p_1, q_1$)
    end if
    if $(p_1, q_1)$ not in the hash table then
      if not dfs1($p_1, q_1$) then return False
    end if
  end for
end for

return True

**end procedure**

**procedure** dfs2($p, q$)

flag($p, q$)

for all successors $q_1$ of $q$ with label $L(p)$ do
  if not accept($q_1$) then continue
  if flagged ($p, q_1$) then return True
  if dfs2($p, q_1$) then return True
  end if
end for

return False

**end procedure**

The typical double DFS algorithm is aimed at finding a path to a cycle containing the accepting state of a buchi automata and can be used to do on-the-fly checking. We adapt it to find an accepting run satisfying LTL formula $F_{q_{bad}}$ in the intersection of a transition system and buchi automata. Procedure “check” returns true if the LTL specification is satisfied. Procedure “dfs1” is used to find an accepting run satisfying LTL formula $F_{q_{bad}}$ and procedure “dfs2” is used to find a cycle containing $q_{bad}$. As every non accepting states in the IIS buchi automata has no outgoing transition, we can ignore them when traversing such states in the product automata since they can not lead to any new state.

In summary, the eager automata-based solution has following advantages over the LTL based approach:
Compared with the lazy proof, it can stop the whole checking process as soon as the LTL specification becomes satisfied.

In comparison to eager proof, as the Büchi automata is constructed incrementally, it is much more efficient than constructing everything from scratch. Besides, the algorithm proposed in [24] consists of two stages: constructing and minimizing. The minimizing stage can also help to boost the performance. The size of the minimized Büchi automata is not monotonically increasing since adding IIS might enable new minimization possibilities leading even to reduction.

Last but not least, as the checking of the LTL specification is implemented in the eager automata-based approach, it can reduce the overhead brought by repeated calling the third-party LTL checkers greatly.

3.5.3 An Illustrative Example

Take the automaton in Fig. 1 for example again. Suppose the given bound is 10. We first construct an initial transition system TS from the graph structure of the LHA, Fig. 3, then the BMC procedure searches for candidate paths incrementally and the first detected IIS path segment is \((v_0) \rightarrow (v_1)\). We construct Büchi automata \(\omega_1\) (shown in Fig. 7) from it and intersect \(\omega_2\) with \(TS\). Then we search for a SCC containing \(q_{v_0}\) and succeed. So we continue the BMC procedure and get another IIS path segment \((v_2) \rightarrow (v_3)\). Then we construct Büchi automata \(\omega_2\) (shown in Fig. 8) based on \(\omega_2\), which can rule out the two detected IIS path segments. At last, we intersect \(\omega_2\) with \(TS\) and find that the product transition system does not have state \(q_{v_5}\), which shows the nonexistence of a counterexample. Then the proof procedure stops the BMC procedure immediately and gives an unbounded result. In this way, we are able to know a general model checking result by searching for a very small state space, which saves a lot of computation time.

4 IMPLEMENTATION AND CASE STUDIES

The LTL-based verification techniques presented in this paper have been implemented into a path-oriented bounded reachability checker of LHA-BACH [10]. For the lazy and eager LTL based solution, we use a typical LTL model checker NuSMV [17] to check the satisfiability of LTL specification. These two different implementations of BACH are marked as BACH (Lazy_NuSMV) and BACH (Eager_NuSMV) respectively. The implementation of the eager automata-based solution is marked as BACH (Automata). In addition, as mentioned in Section 3.3, we also implemented the IC3 based approach into BACH and the underlying IC3 based tool we selected is iimc [26] which supports analysis of LTL specification. We mark the implementation as BACH (IC3).

In order to evaluate the performance of the presented techniques, we carry out extensive evaluations over a set of widely-used benchmarks, which include the aforementioned water-level monitor system in Fig. 1, the temperature control system from [44], the communication based train control system from [16], sample automata in Fig. 9 and an extended version of the navigation benchmark [36]. The navigation example models the motion of a point robot in a n-dimensional cube. The cube is partitioned into \(n^n\) rectangular regions and each such region is associated with a vector field described by the flow equations. In order to increase both discrete and continuous complexity of the model, we use a generalization method introduced in [32] to generate a n-dimensional navigation model. As our work focuses on linear hybrid automata, the flow conditions of the velocity variables in the model are unified as in the range of \([-2, 2]\).

Aside from the above models, we also conduct case studies on a scalable automated highway system from [13] in Fig. 10. It is worth noting that the size of the highway system model can be easily expanded by introducing more cars into the system, which will increase new locations and variables in the model. The target locations, \(q_{\text{target}}\), are all marked by double circles in the models and they are all unreachable.3

3. Due to the space limit, please refer to the tool’s website, http://seg.nju.edu.cn/BACH/ic16, for the detailed models.
4.1 Comparison between Different Proof Approaches

In the last section, we introduce several proof approaches to derive an unbounded proof after or during a BMC checking procedure. In order to evaluate the performance of these approaches, we conduct a series of experiments on the above-mentioned benchmarks.

The experiments are conducted on a ThinkCenter workstation (Intel i5 Quad CPU 3.1 GHz, 8 GB RAM running Ubuntu 13 64 Bit). The time and memory usage limit for the experiment are set to one hour and 4 GB respectively. The value of the bound means the largest number of discrete locations that a path can have in the state space under searching. The executable BACH and the input models we use in experiments for both BACH and SpaceEx are all available from http://seg.nju.edu.cn/BACH/ tc16.

The experiment data for the time and memory usage spent in each benchmark is shown in Table 1. The quickest result is marked in bold. In addition, the number of IIS path segments detected by the underlying LP solver is also given.

It is also worth mentioning that the time spent by BACH means the total time of the BMC and the unbounded proof procedure.

From the data, we can see that:

- All the implementations have consistent results on the benchmarks they can handle. Because the underlying technique is the same, we only change the proof strategy in attempt to improve the performance.

- In general, the performance of BACH (Eager_NuSMV) is the worst among the four implementations. Compared to BACH (Lazy_NuSMV), it needs to perform LTL model checking once a new IIS is located, in other words, call the third-party LTL checker repeatedly, which brings heavy overhead especially when the number of IIS is large. On the other hand, in comparison to BACH (Automata), it has to conduct repeated LTL model checking from scratch, which wastes a lot of time.

- BACH (Automata) outperforms BACH (Lazy_NuSMV) significantly when handling large models. The reason is twofold:

  1. We adopt a dedicated algorithm introduced in [24] to construct a small Büchi automata efficiently. As the last step of LTL model checking is to check the emptiness of the product Büchi, it is easier to do that in a smaller automata. For example, when handling the NAV_5 3 model, the automata based approach can prove that the LTL specification is satisfied in 3 seconds while it takes NuSMV over 40 seconds.

  2. In the automata based solution, the proof procedure can tell the BMC solving process to terminate as long as it finds the LTL specification is satisfied. In contrast, BACH(Lazy_NuSMV) has to wait for the completion of the BMC procedure. Take the LHA motorcade_500 for example, BACH(Automata) is able to prove the target

<table>
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<th>#locs</th>
<th>#vars</th>
<th>#IIS</th>
<th>Time (s)</th>
<th>Mem. (MB)</th>
<th>Time (s)</th>
<th>Mem. (MB)</th>
<th>Time (s)</th>
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</table>

Table 1

Results of Applying Enhanced BACH to Different LHA Cases

- #locs and #vars denote the number of locations and continuous variables in the LHA, respectively. #IIS denotes the number of detected IIS path segments. T.O. is a time out of 3,600 seconds. EXC means the checker threw an exception. When the result is T.O. or EXC, the corresponding blank of memory usage is marked as ‘-’. motorcade_i means automated highway system with i cars. NAV_5 3 denotes the navigation example with three partitions and five dimensions. The bound we set for BMC in BACH is 10 for all models and if BACH gives an unbounded result, corresponding time is marked with subscript U, otherwise, the result is marked with subscript B. Last but not least, all the target locations in the benchmarks are unreachable.
location is not reachable in the unbounded state space when unrolling the model to depth 3 and stop the BMC procedure immediately in 7.43 seconds. Unlike that, BACH(Lazy_NuSMV) has to finish the BMC search at first, which can not be even finished in the one hour time limit due to the huge search space.

As BACH(Lazy_NuSMV) outperforms BACH(IC3) in most of the cases in Table 1, we use BACH(Lazy_NuSMV) to stand for the lazy proof in the experiments. In order to further compare the performance of BACH(Automata) and BACH(Lazy_NuSMV), we also present the experiment data of applying them to analyze models in the benchmark with different bound settings. The result is shown in Table 2, we also list the time needed for the BMC analysis of the same problem as well. From the data we can see that:

- BACH (Automata) outperforms BACH (Lazy_NuSMV) significantly on almost all cases, especially on problems with large bound. In detail, the value of the bound has little influence on the performance of BACH(Automata) while the performance of BACH (Lazy_NuSMV) is greatly affected by it. BACH (Automata) adopts an eager way to check the satisfiability of the LTL specification and it can know the BMC result also stands beyond the bound whenever the collected IIS are strong enough to block the graph. In this case, the proof procedure can stop the BMC solving process to avoid the further meaningless search of the state space. In contrast, BACH (Lazy_NuSMV) has to wait for the completion of the BMC procedure whose performance depends on the value of the threshold heavily.

- Compare with the BMC procedure of BACH, BACH (Lazy_NuSMV) spends a bit more time to finish the whole analysis as it needs to conduct an extra proof besides the BMC checking. On the other hand, BACH(Eager_Automata) outperforms the BMC procedure of BACH in almost all cases, especially when the bound is large. The reason is that BACH(Automata) is able to know the unbounded statement when collecting enough IISes and will terminate to avoid the further meaningless analysis. In contrast, the basic BACH can not know such information in advance and has to search the state space within the bound exhaustively.

### 4.2 Comparison with State-of-the-Art Competitors

We have evaluated the performances of different proof approaches numerically in the last paragraph. In general, BACH(Automata) behaves the best in almost all cases, so we use it to represent BACH. Now, we focus on the comparison of the performance of BACH with the state-of-the-art classical LHA model checker, SpaceEx [5]. According to the documentation, SpaceEx supports two scenarios: PHAVer and Support Function. The former scenario uses the polyhedral-based method to compute the exact reachable state space as the tool PHAVer [4], while the latter adopts support functions [29] to numerically compute the reachable state space. In the experiment, we compare BACH with SpaceEx in both two scenarios, which are marked as SpaceEx (PHA.) and SpaceEx (Supp.), respectively.

The experiment data for the time and memory usage spent in each benchmark is shown in Table 3. From these data, we can see that:

- BACH successfully proved the bounded unreachable results also stand in the unbounded state space for most of the benchmarks, 9/10, except the sample automaton. This confirms that the LTL-based approach in BACH is not complete, however it increases our confidence that many practical cases can be proved to be completely unreachable due to the existence of IIS path segments. In this situation, BACH can produce an unbounded result. BACH only fails to give an unbounded result for the sample automaton. The reason is that in this model the IISes found during the BMC procedure are not able to block the graph.

- Comparing with SpaceEx:

  - As pointed out in the SpaceEx’s documentation and website, the support function scenario, SpaceEx (Supp.), is more suitable to handle hybrid automata with piecewise affine dynamics, for example model with flow condition like \( \dot{x} = Ax + Bu + c \). For the class of LHA

---

**TABLE 2**

Results of Applying BACH(Automata), BACH(Lazy_NuSMV) and Basic BACH to Different LHA Cases

<table>
<thead>
<tr>
<th>System</th>
<th>bound</th>
<th>Time (s)</th>
<th>Mem. (MB)</th>
<th>Time (s)</th>
<th>Mem. (MB)</th>
<th>Time (s)</th>
<th>Mem. (MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BACH(Automata)</td>
<td></td>
<td>BACH(Lazy_NuSMV)</td>
<td></td>
<td>BACH(BMC Procedure)</td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>1,000</td>
<td>0.22(_U)</td>
<td>&lt; 1</td>
<td>0.19(_U)</td>
<td>&lt; 1</td>
<td>0.12(_B)</td>
<td>&lt; 1</td>
</tr>
<tr>
<td></td>
<td>50,000</td>
<td>0.57(_U)</td>
<td>&lt; 1</td>
<td>2.47(_U)</td>
<td>82</td>
<td>2.44(_B)</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>200,000</td>
<td>0.74(_U)</td>
<td>99</td>
<td>27.82(_U)</td>
<td>286</td>
<td>27.46(_B)</td>
<td>286</td>
</tr>
<tr>
<td>tcs</td>
<td>1,000</td>
<td>0.99(_U)</td>
<td>&lt; 1</td>
<td>0.33(_U)</td>
<td>&lt; 1</td>
<td>0.25(_B)</td>
<td>&lt; 1</td>
</tr>
<tr>
<td></td>
<td>100,000</td>
<td>1.02(_U)</td>
<td>49</td>
<td>7.2(_U)</td>
<td>133</td>
<td>7.48(_B)</td>
<td>136</td>
</tr>
<tr>
<td></td>
<td>200,000</td>
<td>1.04(_U)</td>
<td>88</td>
<td>28.16(_U)</td>
<td>257</td>
<td>28.12(_B)</td>
<td>261</td>
</tr>
<tr>
<td>motorcade_20</td>
<td>1,000</td>
<td>0.54(_U)</td>
<td>17</td>
<td>0.04(_U)</td>
<td>79</td>
<td>0.17(_B)</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>50,000</td>
<td>1.9(_U)</td>
<td>94</td>
<td>6.28(_U)</td>
<td>716</td>
<td>5.54(_B)</td>
<td>671</td>
</tr>
<tr>
<td></td>
<td>100,000</td>
<td>2.68(_U)</td>
<td>74</td>
<td>15.47(_U)</td>
<td>1,379</td>
<td>15.11(_B)</td>
<td>1,329</td>
</tr>
<tr>
<td>motorcade_100</td>
<td>1,000</td>
<td>1.87(_U)</td>
<td>33</td>
<td>7.77(_U)</td>
<td>360</td>
<td>2.37(_B)</td>
<td>217</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>6.12(_U)</td>
<td>98</td>
<td>17.79(_U)</td>
<td>2,073</td>
<td>13.9(_B)</td>
<td>1,919</td>
</tr>
<tr>
<td></td>
<td>20,000</td>
<td>10.93(_U)</td>
<td>168</td>
<td>29.51(_U)</td>
<td>3,970</td>
<td>26.7(_B)</td>
<td>3,820</td>
</tr>
</tbody>
</table>
considered in this paper, SpaceEx prefers to use the “PHAVer” scenario, SpaceEx (PHA.), to handle. As we can see from Table 3, the performance of SpaceEx (PHA.) is better than SpaceEx (Supp.) in general when handling the class of LHA considered in this paper. Therefore, we focus on the comparison between SpaceEx (PHA.) and BACH in the following paragraph.

- On small cases, both BACH and SpaceEx (PHA.) finish 4 of the 5 cases efficiently. For example, for most of the models with number of locations and variables smaller than 10, both BACH and SpaceEx can solve them quickly in less than 1 second.

- On the other hand, as the reachability analysis of LHA is undecidable, classical model checking technique do not guarantee to terminate. The experiment shows that SpaceEx (PHA.) times out when dealing with the Temperature Control System model, which has only three continuous variables and five discrete locations. The reason is that the value range of the variables in this model is not closed. Therefore, the classical fix point based computation can not terminate. In contrast, BACH is guaranteed to terminate, as it is BMC-based, and can give an unbounded result in many cases according to the experiment.

- On large cases, BACH has better scalability. Take the automated highway system for example, BACH can solve such system with 200 cars, 202 locations and 200 continuous variables, in less than 70 s, while SpaceEx can only handle the system with five cars within the 1 h time limit. As the polyhedra based computation is sensitive to the number of continuous variables in the LHA, the performance of SpaceEx degrades greatly when the size of the model increases. Different from SpaceEx, BACH conducts lightweight BMC checking first. Then, BACH utilizes the IIS found during BMC process to derive an unbounded result using LTL model checking which is a very mature and efficient technique.

5 Related Work

Reachability analysis of linear hybrid automata is a very important problem. Classical model checking techniques [3], [4], [5] try to compute the complete reachable state space of LHA. The basic idea is to compute iteratively the reachable state space of the next-step based on the current state space. The algorithm terminates if the reachable state space of the next-step is contained by the current state space. However, such termination is not guaranteed. Furthermore, as the computation is very sensitive to the number of continuous variables, the state-of-the-art tools developed based on this technique do not scale well to the size of practical problems.

On the other hand, inductive invariant generation has been used in the safety/reachability analysis of hybrid system. In [38], the authors propose to use a function of state called barrier certificate to separate the unsafe region from all possible trajectories starting from a given set of initial conditions. However, this method is difficult to handle system with mixed equations and inequalities [39]. Inductive invariant generation of hybrid system has been studied in the area of theorem proving as well. In study [39], the differential invariants are computed as fixed points by differential logic for hybrid systems in an interactive way by KeYmaera.

In study [28], McMillan proposed an interpolation based approach for unbounded model checking of finite-state transition system. They proposed to compute an over-approximation of the forward reachable state space using SAT-based interpolation. Interpolation has been widely applied in different areas like software checking [42], [43]. It has been extended to nonlinear hybrid system in [40] as well. Craig interpolation (CI) plays the key role in these studies. It remains an important problem to study the scalable approach for CI synthesis for large and complex system [41].

Recently, bounded model checking [6] has attracted a lot of attention as an alternative technique to the classical model checking. The basic idea of this approach is to search for a counterexample whose length is bounded by some integer k in model executions. However, the result of BMC verification only covers the bounded behavior of the LHA model. It remains a very interesting problem that whether we can acquire an unbounded proof from a BMC procedure.

Sheeran et al. proposed an induction based approach called k-induction [21] to check the safety property of finite-state transition system. The idea of this technique is to prove that if a set of states is not reachable in k step, then it is completely not reachable. As hybrid automata contains both discrete and continuous behavior, such approach can not be directly applied. In [22], Moura et al. proposed to use k-induction to verify timed and hybrid automata and they...
generalized the simple path condition used in original $k$-induction to simulate relations. As the implementations of the work are not available now, we cannot compare the performance between our approach and [22] numerically. However, as the unbounded analysis of [22] is conducted by performing induction on continuous dynamics of LHA and the procedure of generating strengthened invariants requires quantifier-elimination, it demands a high computational complexity, which greatly restricts the size of the problem that can be solved.

There are also works focusing on CEGAR of LHA. Study [32] presented a hybrid abstraction based CEGAR loop for the class of rectangular hybrid automata, which is a special subclass of the LHA considered in this paper. Study [37] utilized the counterexample path to synthesize good parameters during the verification in a similar path-oriented way, while the counterexample in our work is used to refine the graph of the LHA.

Study [13] proposed a CEGAR method, called “Iterative Relaxation Abstraction”, for LHA by dropping variables from original LHA in each iteration, and asked PHAVer to solve the simplified model. Since this technique still relies on PHAVer, in other words, geometric computation, as the underlying checker for the abstract model, the computation burden is still heavy. Although the implementations of this work are not available as well, this work [13] also reported its performance on the motorcade series problem. According to [13], the largest motorcade model it solved has 19 variables included. It took [13] 652.51 seconds to solve such a problem, while for a larger problem, motorcade_20, our tool BACH solved it in only 0.71 seconds. Furthermore, BACH solved a much larger system motorcade_500 in only 7.43 seconds successfully.

In [34], Fehnker et al. proposed to locate “cut set” path segments in the graph structure of the LHA model and then validate each cut set one by one in the original model. If a cut set was confirmed to be invalid in the original model, there does not exist a behavior in the LHA that connects the initial location to the target. Hence, the target location is not reachable. Basically, the cut set path segments used in [34] are certain key path segments which are shared by many paths on the discrete graph level. Therefore, they have to validate such cut set path segments in the original LHA model. While in this paper, we use IIS technique to analyze the continuous elements of the path to locate the infeasible path segment core after one path is proved to be infeasible on the continuous level by LP solver. Then, such IIS path segments, which are definitely infeasible, can be removed from the graph structure to benefit our path-enumerating based BMC procedure. Clearly, the IIS path segment used in this paper could be any path segment in the graph structure and hence is different from the cut set path segment in [34].

There are also related works which manipulate infeasible path segments to block the target node in different area. In study [24], Segelken proposed an $\omega$-automata based approach for CEGAR [14] verification of step-discrete linear hybrid models. In each CEGAR iteration, they present the spurious counterexample path found in the last iteration as an LTL formula which is similar to our work. After that, in order to exclude such spurious counterexample path in the refinement of the abstract model, the author proposed an incremental algorithm to translate the corresponding LTL formula to an $\omega$-automata using a dedicated algorithm which is more efficient than the general LTL-to-Büchi translation algorithm [25]. Then the refinement will be done by computing the Cartesian product of the $\omega$-automata and the abstract model. If the product automata satisfies the safety property, then the original model is safe. This efficient LTL-to-Büchi translation algorithm is adopted in our automata based proof to achieve better performance.

On the other side, the difference between this paper and [24] mainly dwells in the following points. First, the counterexample in their CEGAR iteration is identified by LTL model checking on the discrete state space of the step-discrete linear hybrid model. In contrast, the counterexample in our work is given by the BMC procedure of the continuous behavior of the LHA model. Another main difference is that, in the LTL specification checking, we propose an on-the-fly algorithm to check whether the product automata satisfies the LTL formula $F(q_{\text{init}})$, while they adopt a commonly-used approach which has to compute the intersection first.

A similar idea has been proposed in software verification. In [33], Heizmann proposed to compute appropriate abstractions of programs based on statements as alphabet atoms in an automata framework and block inconsistent path segments on the automata level. This idea shares similar merits with us. However, they focus on finding the right abstraction of a program for a given property to dismiss a set of infeasible paths, while our work locates the minimal infeasible path segments directly. It would be an interesting topic to take advantage of static analysis techniques and interpolation generation to refine the infeasible path segments and automata in our future work.

6 Conclusion

Reachability analysis of linear hybrid automata is very difficult. Classical full state space reachability verification is not scalable, while bounded model checking is much more efficient to conduct but the result only covers the bounded behavior of the model. During path-oriented BMC of LHA, a set of infeasible path segments can be located by IIS technique. The target location is completely not reachable when it can not be reached without going through such infeasible path segments.

In this paper, we propose an LTL-based solution to derive an unbounded proof from the BMC procedure by proving whether the target location is blocked by the infeasible path segments located during the BMC checking. We implemented the proposed techniques into a bounded LHA checker BACH. The experiments show that, although this approach is not complete, BACH can give an unbounded result for most of the benchmarks successfully, efficiently and also with a good scalability.

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